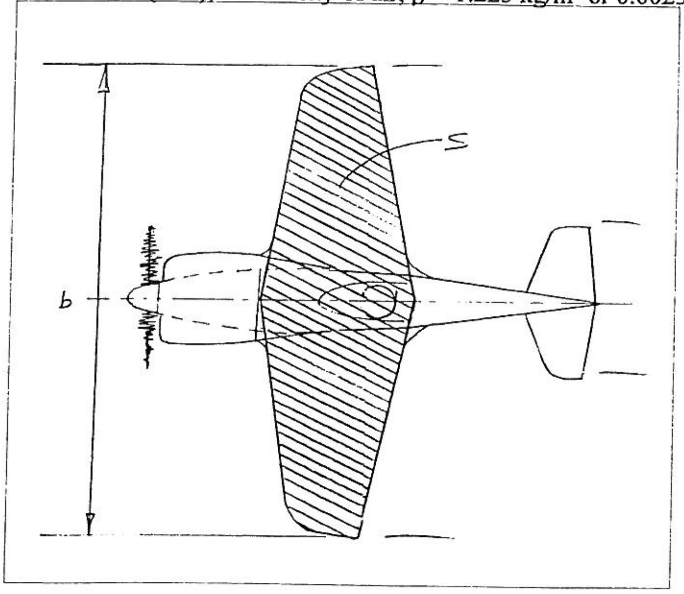
## DERIVATION OF THE AIRCRAFT SYMMETRIC FLIGHT ENVELOPE.
## JAR VLA REQUIREMENTS

The 'symmetric' manoeuvre and gust envelopes form the basis of the in-flight stressing cases for a given aircraft design. The manoeuvre envelope gives the range of speeds and pilot induced 'g' loadings which the aircraft will be required to withstand. The aircraft is considered in 'symmetric' flight, that is, not side-slipping, rolling or yawing. Just pitching and balanced turning manoeuvres are considered. The gust envelope gives the range of 'g' loadings generated by gusts at various speeds which the aircraft will also have to be designed to withstand. These symmetric flight load cases dictate most of the design of the aircraft structure. Combining these two envelopes yields the aircraft flight envelope. The requirements for the flight envelope are contained in **JAR VLA 333 - 345** for VLA aircraft. An example flight envelope is developed in this data sheet for a JAR VLA aeroplane. The 'g' loadings generated **are limit load cases**. The design example given here is for a conventional aircraft. Aircraft which have wings with large sweep angles or abnormally low aspect ratios (less than 4.0) will require different methods of calculating the wing 3-D lift-curve slope. Two cases will be considered, one using metric, and one using imperial units. Relevant imperial equivalents are given in brackets.

REQUIRED DATA. The data which you will require to perform these calculations is as follows:

| **Parameter** | **Symbol** | **Notes** |
|---|---|---|
| Aircraft Max All Up Weight | W | Metric weight is measured in Newtons (N) and is equal to mass (kg) x gravitational acceleration (9.81 $m/s^2$)<br>Imperial weight and force are given in pounds (lb). The imperial unit for mass is the slug. Slugs = pounds/32.185 |
| Wing Area | S | (sq. meters or sq.feet) |
| Wing Span | b | (meters or feet) |
| Wing Maximum Positive Lift Coefficient | $C_{Lmax}$ | Max positive. Use 1.35 if no other information available. |
| Wing Maximum Negative Lift Coefficient | $C_{Lmin}$ | Max negative. Use -1.35 if no other information available. |
| Wing lift-curve slope | a | For the 3-D wing corrected for aspect ratio (calculation method is given here) |

At sea level (ISA), the density of air, $\rho = 1.225\ kg/m^3$ or $0.002376\ slug/ft^3$



Note that:

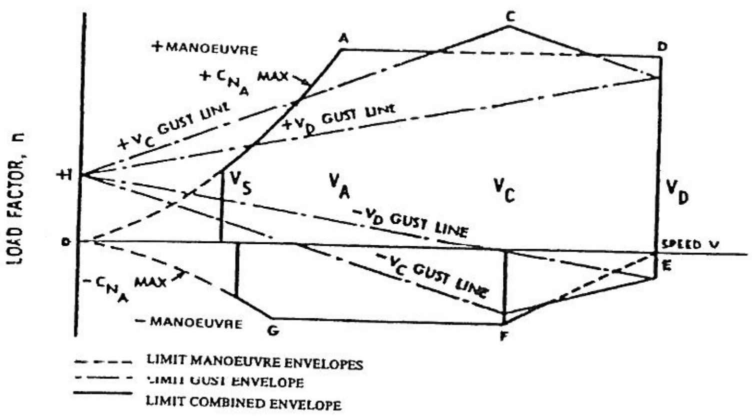
b Span is total span of wing
S Wing area is total wing area, including region inside fuselage, but excluding wing fairings.

CARE MUST BE TAKEN TO USE CONSISTENT UNITS, ie,

METERIC UNITS $g = 9.81\ m/s^2$, Mass measured in kilograms. Force measured in Newtons. Length measured in metres.

IMPERIAL UNITS $g = 32.185\ ft/s^2$ Mass measured in slugs. Weight and force measured in pounds. Length measured in feet.

Flight envelope



NOTE: Point G need not be investigated when the supplementary condition specified in JAR-VLA 369 is investigated.

Both the manoeuvre and gust envelopes are calculated with the aircraft at its projected maximum all-up-weight. The exception to this would be an aerobatic aircraft where both the maximum aerobatic weight and maximum take-off weight would have to be considered.

Sketched above is an example flight envelope. The solid lines show the manoeuvre envelope. The dashed lines show the gust envelope. The curve O-A is the positive stall curve. At 1g, the minimum speed at which the aircraft can sustain level flight is the aircraft 'stall speed', $V_S$. As the 'g' increases due to gusts or manoeuvring, the stall speed increases. At the positive limit load factor, $n_1$, (3.8 g for most VLA aeroplanes), the aircraft will stall at speed $V_A$, the aircraft design manoeuvring speed. Curve O-G is the negative stall curve. $V_G$ is the stall speed of the aircraft at the negative limit load factor, $n_2$ (-1.5 g for most VLA aeroplanes). $V_C$ is the design cruise speed, the minimum value of which is calculated from the VLA requirements. $V_C$ is an important parameter, and is the essential basis for your diagram. Provided that $V_C$ is equal or higher than the minimum required by JAR VLA, it is whatever *you* decide. The designer may want $V_C$ to be as low as possible to limit the loads on the airframe. The pilot will want $V_C$ to be high to give him scope to throw the aeroplane about. $V_D$ is the design diving speed which is dependent on $V_C$ It would therefore pay not to be too generous with your value for $V_C$, or you will have an onerous design diving speed to content with However, you should have a large enough margin to ensure that you will not approach $V_{NE}$ during normal manoeuvring. $V_D$ is not the $V_{NE}$ of the aircraft. Usually, $V_{NE} = 0.9\ V_D$. One further speed to define is $V_H$, the maximum steady level flight speed.

| Speeds | | |
|---|---|---|
| | $V_S$ | Computed stall speed with flaps retracted |
| | $V_{S.1g}$ | Computed stall speed at 1g |
| | $V_{S\text{-}1g}$ | Computed stall speed at -1g |
| | $V_A$ | Design manoeuvring speed |
| | $V_C$ | Design cruising speed |
| | $V_D$ | Design dive speed |
| | $V_G$ | Design manoeuvring speed (negative load case) |
| | $V_H$ | Maximum steady level flight speed |

CALCULATION OF MANOEUVRE ENVELOPE

$$V_{S.1g} = \sqrt{\frac{Mg}{\frac{1}{2}\rho S C_{L_{\max}}}}$$ the 1 g stall speed

At other values of n (positive load factor), the stall speed is given by $V_S = V_{S.1g}\sqrt{n}$ where n will vary between 0 and $n_1$ (3.8 g). This will give the positive stall curve of the flight envelope.

$$V_A = V_S\sqrt{n_1}$$ the positive design manoeuvring speed (VLA 335(c))

$$V_{S.-1g} = \sqrt{\frac{-Mg}{\frac{1}{2}\rho S C_{L_{\min}}}}$$ the -1g stall speed.

At other values of n (negative g loading), the stall speed is given by $V_S = V_{S.-1g}\sqrt{-n}$ where n will vary between 0 and $n_2$ (-1.5 g). This will give the negative stall curve of the flight envelope.

$$V_G = V_{S.-1g}\sqrt{-n_2}$$ the negative design manoeuvring speed

The design cruising speed $V_C$ may not be less than 2.4 $(Mg/S)^{1/2}$ metres/sec or 4.7 $(Mg/S)^{1/2}$ knots (VLA 335(a)(1))

Note that these formulae for the minimum values of $V_C$ assume that you are using metric units. An imperial equivalent is given in the example. You are not constrained to use these minimum values. Indeed, if your aircraft has high performance in terms of a high cruising speed, the value of $V_C$ would be much greater than the minimum. Additionally:

It is recommended that $V_C$ should not be less than 0.9 $V_H$ (VLA 335(a)(2))

If you have chosen $V_C$ to be above the minimum allowable then:
$V_D$ may not be less than 1.25 $V_C$ (VLA 335(b)(1))

If $V_C$ is selected as the minimum allowed by JAR VLA then:
$V_D$ may not be less than 1.4 $V_C$ (VLA 335(b)(2))

EXAMPLE

An aircraft has a wing area of 9.29 m$^2$ (100 sq.ft), a span of 7.315 m (24 feet), and a maximum all-up-weight of 453.6 kg (1000 lb). The aerofoil section used is NACA 2415.

Although we know the maximum lift coefficient of the 2-D NACA 2415 aerofoil section, taken from ref.1, we do not know what the 3-D maximum lift-coefficient of the wing will be. This value will be less that the 2-D value because of tip-losses, etc. We will therefore take $C_{Lmax} = 1.35$ and $C_{Lmin} = -1.35$, as recommended by JAR VLA A9(b)(1)(ii).

For a VLA aircraft, $n_1 = 3.8$ g and $n_2 = -1.5$ g (VLA 337(a) and (b))

METRIC SOLUTION

$$V_{S.1g} = \sqrt{\frac{453.6 \times 9.81}{\frac{1}{2} \times 1.225 \times 9.29 \times 1.35}} = 24.1 \text{ m/s}$$

$$V_A = 24.1\sqrt{3.8} = 46.9 \text{ m/s}$$

$$V_{S.-1g} = \sqrt{\frac{-1 \times 453.6 \times 9.81}{\frac{1}{2} \times 1.225 \times 9.29 \times -1.35}} = 24.1 \text{ m/s}$$

$$V_G = 24.1\sqrt{-(-1.5)} = 29.5 \text{ m/s}$$

$$V_{C_{min}} = 2.4\sqrt{\frac{453.6 \times 9.81}{9.29}} = 51.5 \text{ m/s}$$

$$V_D = 1.4 \times 51.5 = 72.1 \text{ m/s}$$

To obtain speeds in miles per hour multiply m/s by 2.237

Hence $V_{S\,1g}$ = 53.8 mph
$V_A$ = 105.0 mph
$V_G$ = 66.0 mph
$V_C$ = 115.2 mph
$V_D$ = 161.2 mph

THE POSITIVE STALL CURVE

To obtain speeds in miles per hour multiply m/s by 2.237

$$V_S = 24.1\sqrt{n}$$

| n | $V_s$ (m/s) | $V_s$ (mph) |
|---|---|---|
| 0 | 0.0 | 0.0 |
| 0.5 | 17.0 | 38.0 |
| 1.0 | 24.1 | 53.9 |
| 1.5 | 29.5 | 66.0 |
| 2.0 | 34.0 | 76.1 |
| 2.5 | 38.1 | 85.2 |
| 3.0 | 41.7 | 93.3 |
| 3.8 | 46.9 | 104.9 |

THE NEGATIVE STALL CURVE

To obtain speeds in miles per hour multiply m/s by 2.237

$$V_S = 24.1\sqrt{-n}$$

| n | $V_s$ (m/s) | $V_s$ (mph) |
|---|---|---|
| 0 | 0.0 | 0.0 |
| -0.5 | 17.0 | 38.0 |
| -1.0 | 24.1 | 53.9 |
| -1.5 | 29.5 | 66.0 |

(The negative stall curve would be different from the positive stall curve if $C_{lmax}$ was different to $C_{Lmin}$)

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DESIGN DATA SHEET/ 1

## IMPERIAL SOLUTION

$$V_{S1g} = \sqrt{\frac{1000}{\frac{1}{2} \times 0.002376 \times 100 \times 1.35}} = 79.0 \text{ ft/s}$$

$$V_A = 79.0\sqrt{3.8} = 154.0 \text{ ft/s}$$

$$V_{S-1g} = \sqrt{\frac{-1 \times 1000}{\frac{1}{2} \times 0.002376 \times 100 \times -1.35}} = 79.0 \text{ ft/s}$$

$$V_G = 79.0\sqrt{-1.5} = 96.7 \text{ ft/s}$$

$$V_{C_{min}} = 53.4\sqrt{\frac{1000}{100}} = 168.9 \text{ ft/s}$$ (the 53.4 constant is used for imperial units)

$$V_D = 1.4 \times 168.86 = 236.4 \text{ ft/s}$$

To obtain speeds in miles per hour multiply ft/s by 0.6818.

Hence

| | | |
|---|---|---|
| $V_{S1g}$ | = | 53.8 mph |
| $V_A$ | = | 105.0 mph |
| $V_G$ | = | 66.0 mph |
| $V_C$ | = | 115.2 mph |
| $V_D$ | = | 161.2 mph |

THE POSITIVE STALL CURVE

To obtain speeds in miles per hour multiply ft/s by 0.6818

$$V_S = 79.0\sqrt{n}$$

| n | $V_s$ (ft/s) | $V_s$ (mph) |
|---|---|---|
| 0 | 0.0 | 0.0 |
| 0.5 | 55.9 | 38.0 |
| 1.0 | 79.0 | 53.9 |
| 1.5 | 96.8 | 66.0 |
| 2.0 | 111.7 | 76.1 |
| 2.5 | 124.9 | 85.2 |
| 3.0 | 136.8 | 93.3 |
| 3.8 | 154.0 | 104.9 |

THE NEGATIVE STALL CURVE

To obtain speeds in miles per hour multiply ft/s by 0.6818

$$V_S = 79.0\sqrt{-n}$$

| n | $V_s$ (ft/s) | $V_s$ (mph) |
|---|---|---|
| 0 | 0.0 | 0.0 |
| -0.5 | 55.9 | 38.0 |
| -1.0 | 79.0 | 53.9 |
| -1.5 | 96.8 | 66.0 |

(The negative stall curve would be different from the positive stall curve if $C_{lmax}$ was different to $C_{Lmin}$)

## CALCULATION OF GUST ENVELOPE

The gust envelope is derived assuming vertical gusts acting on the aircraft of

15.24 metres/sec (50 feet/sec) at $V_C$ (VLA 333(c)(1)(i))
7.62 metres/sec (25 feet/sec) at $V_D$ (VLA 333(c)(1)(ii))

The gust load factors at these speeds may be computed as follows:

$$n = 1 + \frac{\frac{1}{2}\rho_0 V a K_g U_{de}}{Mg/S} \qquad \text{(VLA 341)}$$

where $K_g = \dfrac{0.88\mu_g}{5.3 + \mu_g}$ gust alleviation factor $\qquad \mu_g = \dfrac{2(M/S)}{\rho \bar{C} a}$

$$\bar{C} = S/b$$

$U_{de}$ = derived gust velocities referred to above
$\rho_0$ = density of air at sea level
$\rho$ = density of air at design point
V = aircraft speed
a = 3-D lift curve slope

The 3-D lift-curve slope can be estimated from the approximate relationship $a = \dfrac{a_1^0 A}{2 + \sqrt{4 + A^2}}$

where $a_1^0$ is the 2-D aerofoil lift-curve slope obtained from sources such as ref.1.

$A$ is the wing aspect ratio where $A = \dfrac{b^2}{S}$

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DESIGN DATA SHEET/ 1

## GUST ENVELOPE EXAMPLE

### METRIC UNITS

The 2-D lift-curve slope of the NACA 2415 was measured as 0.106 per degree from ref.1. This was achieved by finding the gradient of the linear portion of the lift vs angle-of-attack curve for this section at a Reynolds number of 3,000,000 (typical).

Aspect ratio $A = \frac{7.315^2}{9.29} = 5.76$ $\qquad \overline{C} = \frac{9.29}{7.315} = 1.27$ m

3-D lift-curve slope, $a = \frac{0.106 \times 5.76}{2 + \sqrt{4 + 5.76^2}} = 0.0754$ per degree = 0.0754 x 180/$\pi$ = 4.32 per radian.

$$\mu_g = \frac{2 \times 453.6 / 9.29}{1.225 \times 1.27 \times 4.32} = 14.53 \qquad K_g = \frac{0.88 \times 14.53}{5.3 + 14.53} = 0.644$$

$$n = 1 + \frac{\frac{1}{2} \times 1.225 \times V \times 4.32 \times 0.644 \times U_{de}}{453.6 \times 9.81 / 9.29}$$

GUST ENVELOPE

| | V (m/s) | $U_{de}$ (m/s) | n |
|---|---|---|---|
| $V_C$ | 51.5 | +15.24 | +3.79 |
| $V_C$ | 51.5 | -15.24 | -1.79 |
| $V_D$ | 72.1 | +7.62 | +2.95 |
| $V_D$ | 72.1 | -7.62 | -0.95 |

### IMPERIAL UNITS

Aspect Ratio $A = \frac{24^2}{100} = 5.76$

3-D lift-curve slope, $a = \frac{0.106 \times 5.76}{2 + \sqrt{4 + 5.76^2}} = 0.0754$ per degree = 0.0754 x 180/$\pi$ = 4.32 per radian.

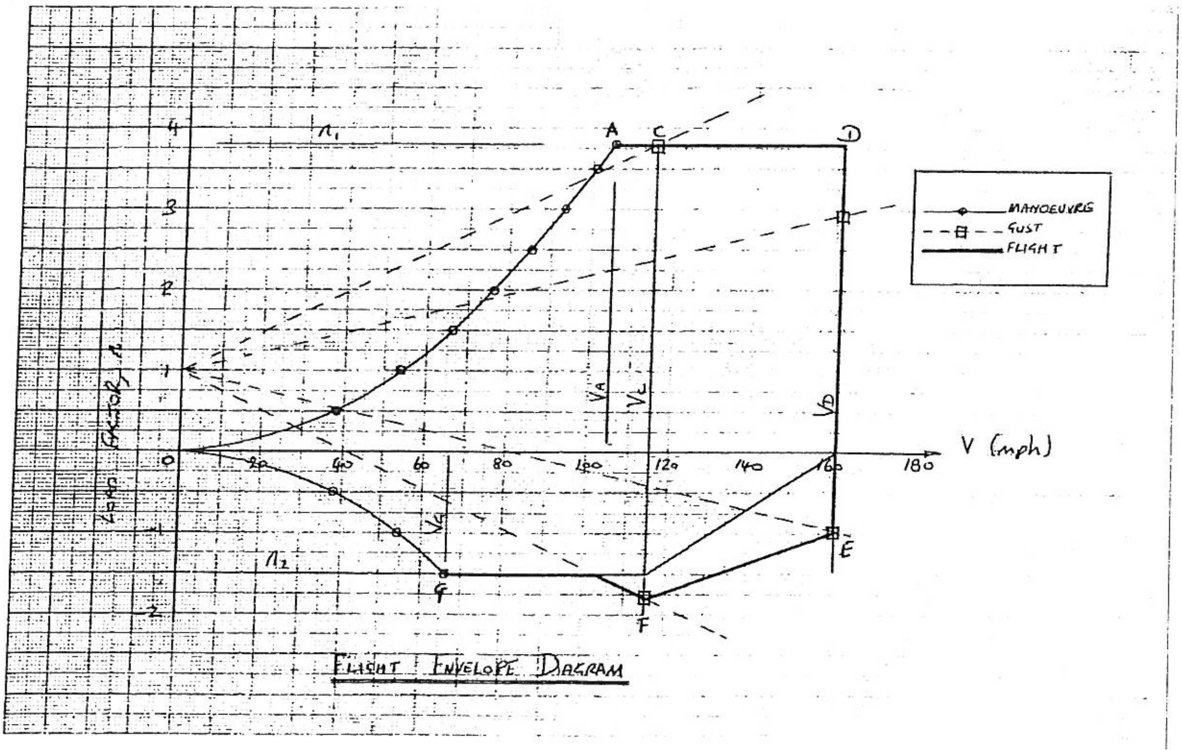
$$\overline{C} = \frac{100}{24} = 4.17 \text{ ft} \qquad \mu_g = \frac{2 \times \left(\frac{1000}{32.185}\right) / 100}{0.002376 \times 4.17 \times 4.32} = 14.53 \quad K_g = \frac{0.88 \times 14.53}{5.3 + 14.53} = 0.644$$

$$n = 1 + \frac{\frac{1}{2} \times 0.002376 \times V \times 4.32 \times 0.644 \times U_{de}}{1000 / 100}$$

GUST ENVELOPE

| | V (ft/s) | $U_{de}$ (ft/s) | n |
|---|---|---|---|
| $V_C$ | 168.9 | +50 | +3.79 |
| $V_C$ | 168.9 | -50 | -1.79 |
| $V_D$ | 236.4 | +25 | +2.95 |
| $V_D$ | 236.4 | -25 | -0.95 |

## FLIGHT ENVELOPE



FLIGHT ENVELOPE DIAGRAM

Note that the gust envelope development lines pass through 1'g' at V=0 mph. This is because it is assumed that the aircraft is accelerated by each gust from the 1'g' level flight condition.

REQUIRED REPORT.

The above workings are to be presented, plus the resultant flight envelope diagram.

It is important to note that the wing loading on the aircraft will be the flight load derived above + the tail down-load - the wing weight.

REFERENCES

1. Abbot I & Von Doenhoff A. **Theory of Wing Sections** Dover Publications, New York, USA 1958

2. JAR-VLA Very Light Aeroplanes C.A.A., Printing and Publication Services Greville House, 37 Gratton Road, Cheltenham, Glos. GL50 2BN